The Concept of Inflation

Introduced by Alan Guth, circa 1980, to provide answers to the following 5 enigmas:

1. “horizon problem”. How come the cosmic microwave background radiation is so uniform in very different parts of the sky that apparently are NOT causally connected (ie, are outside each other’s light cone)?

2. “flatness problem”. Why does the Universe have a mass-energy density that leaves it so close to topological flatness?

3. “origin of structure”. For galaxies to form, there must have been substantial fluctuations in the primordial Universe. Why did they exist?

4. “singularity at t=0”. If we extrapolate backward in time from current conditions, we arrive at the condition of infinite density and temperature at the beginning of time, t=0. How can these infinities be avoided?

5. “uniqueness of the Universe”. If there is only one universe with a precise beginning, what was going on before that?
Mathematician’s view of inflation

Add one term to Friedmann Eq.

\[ V^2 - \frac{8\pi G}{3} \rho R^2 - \frac{\Lambda}{3} R^2 = \text{constant} \]

\( V^2 \) \( \rho \) energy in scalar field

Consider a new case:
- “flat” universe; i.e., curvature constant is ZERO
- cosmological constant \( \Lambda \) term DOMINATES over ordinary matter term

Then things simplify to:

\[ V^2 = \frac{\Lambda}{3} R^2 \]

Using calculus, this relation has a simple solution:

\[ R = C \sqrt[3]{\Lambda} \]

exponential increase of radius \( R \) with time \( t \)

\[ \to \text{INFLATION} \]

Scale \( R \) doubles in size every \( 10^{-35} \) seconds for some 100 doublings (until \( t = 10^{-33} \) sec)

\[ 2^{100} = 10^{30} \]

\( \to \text{scale of atom inflated to scale of galaxy}!! \)

Relative sizes in alternative models of the expanding universe

<table>
<thead>
<tr>
<th>Model</th>
<th>Radiative</th>
<th>Matter-dominated</th>
<th>Free expansion</th>
<th>Inflation</th>
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<tbody>
<tr>
<td>( Rax )</td>
<td>( t^{1/4} )</td>
<td>( t^{4/3} )</td>
<td>( t^3 )</td>
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<td>8.0</td>
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</tbody>
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256t

256t
Physicist’s view of (chaotic) inflation

* There can be energy density associated with a scalar field but NO ordinary matter
  → space SEEMS empty (ie, a vacuum)
  → Call this a “false vacuum” (it seems empty but it isn’t)

* Virtual particles & antiparticles can be created in the false vacuum, then annihilate

* There can be fluctuations in this false vacuum field. Occasionally, by chance in the infinity of time and space, the energy density in a fluctuation can be comparable to the Planck mass (10^{19} \text{ GeV}) and be sufficiently uniform on the Planck scale (10^{-33} \text{ cm}).

Interesting web page: http://www.stanford.edu/~alinde
Inflation (continued)

# The region inflates following the mathematic imperative of the Friedmann Eqn.
The vastly expanded region becomes topologically flat; k=0. The energy density of the scalar field is constant during the expansion.

# The inflation is broken when the energy density of the scalar field is transferred over to ordinary matter. The density in ordinary matter corresponds to the requirement of the flat universe; "critical density" for a marginally closed universe.

\[(V/R)^2 - (8\pi/3)G\rho - \Lambda/3 = 0\]

# Particles annihilate to release energy. --> universe heated to the high temperatures of the Big Bang: temperatures and densities slightly below the Planck density values, of order the Grand Unified Theory (GUT) values.

# The universe passes through the GUT "phase transition", \(X\overline{X}\) annihilate, excess of matter over antimatter established.

# With the concept of chaotic inflation, a new inflation can occur spontaneously. The probability can be incredibly small but would happen occasionally in the infinity of space and time.

# Variations in the scalar field energy level driving inflation can lead to variations in the symmetry-breaking that defines what we call forces (gravity, electromagnetic, weak, strong) so these could be different in other "universes". Possibly our 3 dimensions + time are a random happenstance of our inflation and other dimensionalities are possible.

5 enigmas

1. **Horizon problem.** Places that seem not causally connected because they are so far apart today were very close before the period of inflation - so close that they were in thermal equilibrium.

2. **Flatness problem.** A small space expanded to a large space seems close to flat, just as the surface of the Earth seems flat to us. The density associated with almost flat space is the `critical` density.

3. **Origin of structure.** Small statistical fluctuations that must inevitably exist in an ensemble of particles are amplified by the inflation. Theory predicts a spectrum of fluctuations similar to what is observed.

4. **Singularity at t=0.** There never was an infinity of density or temperature in the inflation model. The conversion of energy out of the scalar field creates particles with temperatures and densities near the GUT regime and the Universe cools and dilutes from there.

5. **Uniqueness of the Universe.** With chaotic inflation there can be, and probably are, other disconnected universes with their own space and starting time and possibly very different physical properties.

Fluctuations arise out of the initial inflation with similar amplitudes on all scales. However, subsequently, smaller scale fluctuations enter the horizon earlier so have longer to grow from self-gravity. So the densities on small scales are greater and structure is formed with small objects first then merging into large objects.
Key Point 1

Birth of the Big Bang

Expansion rate  Normal matter slows  Vacuum energy accelerates

\[ \frac{V^2}{R^2} - \frac{8\pi G\rho}{3} - \frac{\Lambda}{3} = 0 \]

Inflation is initiated when vacuum energy totally dominates matter. Space filled with vacuum energy expands exponentially. Inflation is terminated when energy from the vacuum is transformed into matter.

Matter-antimatter then in equilibrium with radiation with the conditions associated with the universe at the GUT era at $10^{-35}$ seconds.

Small difference in matter - antimatter ratio develops. The universe expands at a slowing rate due to gravity and cools.
Key Point 2

Statistical variations in the vacuum energy virtual particle fluctuations are entrained in the exponential expansion of space. Tiny fluctuations become huge.

As the vacuum energy is transferred into particles the variations in the vacuum energy are frozen in as local variations in the density of matter.

These variations have no preferred scale (big and small equally likely).

Light travel time across small scale variations is shorter than for large variations so small perturbations become causally connected (enter the horizon) earlier than large perturbations. => Small perturbations have more time to grow through gravitational self-attraction.

=> Small scale perturbations collapse first; larger objects form later through coalescence of smaller objects.
Greg’s Ultimate Unified Theory of Cosmology

Stretch!
I command you!

Big Bang Expansion
13.7 billion years
3. Inflation: vacuum energy drove an exponential increase in space which was only broken when the energy in the vacuum transferred to particles. The concept of vacuum energy is a familiar part of modern physics and we now find that something similar, dark energy, dominates the current energy budget of the universe.

4. Onset of inflation: an extreme high vacuum energy density was required to initiate inflation. There is speculation regarding how such a condition could arise and involves the possibility that it could arise often and that there could be an infinity of universes.

1. Big Bang: the universe was once a hot and dense plasma. The physics since a time of a microsecond that resulted in the formation of H and He nucleons is well understood.

2. Baryogenesis: matter in the universe arose out of a 1 part in a billion imbalance between particles and antiparticles. Non-conservation of particles could have arisen during the GUT era, $t \approx 10^{-35}$ seconds, through understood but poorly constrained processes.